Oscillation of Second Order Sub linear Neutral Differential Equations with Distributed Deviating Arguments

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Abstract

In this paper, we obtain new sufficient conditions for oscillation of solutions of second-order subline arneutral differential equations with distributed deviating arguments. Using Riccatitrans formation and comparison technique, we attain new oscillation criteria that achieve to few of the results proclaimed in this research. Examples are presented to illustrate the significance of the main results.

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1. Introduction

This paper concerned with generating new criteria for the oscillation of second-order differential equation with sub linear neutral term and distributed deviating arguments

$$\left(\psi(\ell)(z'(\ell))^{\beta}\right)' + \int_a^b q(\ell,s)x^{\delta}\left(\sigma(\ell,s)\right)ds = 0 \; (E)$$

where $\ell \in \mathrm{I} = [\ell_0, \infty)$, and

$$z(\ell) = x(\ell) + \int_c^d p(\ell,s) x^{\delta}\big(\tau(\ell,s)\big) ds = 0.$$

Throughout, we consider that

(H_1) α, β and δ are ratios of odd positive integers with $0 < \alpha \leq 1$;

(H_2) $\psi \in C(I,(0,\infty)), p \in C(I \times [c,d],(0,\infty)), q \in C(I \times [a,b],(0,\infty))$ and

$$\int_{\ell_0}^{\infty} \psi^{-\frac{1}{\beta}}(s)\, ds = \infty; \tag{1.1}$$

(H_3) $\tau, \sigma \in C(I,\mathbb{R}), \tau(\ell,s) \leq t, \sigma(\ell,s) \leq t$ and $\lim_{t\to\infty} \tau(\ell,s) = \lim_{t\to\infty} \sigma(\ell,s) = \infty$;

By a solution of (E),which means a function $x \in C^{'}([\ell_x,\infty),\mathbb{R}), \ell_x \geq \ell_0$,whichhas the property $\psi(\ell)(z'(\ell))\beta \in C'([\ell_0,\infty),\mathbb{R})$, and satisfies (E) on $[\ell_x,\infty)$.We study only those solutions x of (E) such that $sup\{|x(\ell)|: \ell \geq \ell_x\} > 0$ for all $\ell \geq \ell_x$.If x has infinitely many zeros in I, then x is called oscillatory; otherwise nonoscillatory. Equation (E) is called oscillatory if all its solutions are oscillatory.

Neutral type delay differential equations not only have theoretical importance but also they have great practical importance. In fact these equations appear in the study of vibrating masses attached to an elastic bar, in the solution of variational problems with time delays, and problems involving electric networks consist of loss less transmission lines, see[7,8].

In the past, several years, the problem of investigating the oscillation conditions of neutral type delay differential equations has been a very elective research area, see[1,7,11,15].In recent years in [6,14] the authors considered the equation

$$(\psi(\ell)(z'(\ell))\beta)' + f(\ell)x^{\delta}(\sigma(\ell)) = 0, \ell \geq \ell_0 \tag{E_1}$$

where $z(\ell) = x(\ell) + p(\ell)x^{\alpha}(\tau(\ell))$,and obtained criteria for the oscillation of (E_1) in the case $0 < \alpha \leq 1$.But so far no oscillation result available for equation of the type (E).This observation inspiring us to investigate the oscillatory behavior of solutions of (E). Thus, the results contained in this paper are new and complement to that of in [2,4–6,12,13,15,16,18].

2. PRELIMINARYLEMMAS

For easy reference, we denote that

$$E(\ell) = \int_a^b q(\ell,s)\left[1 - \left(\alpha + \frac{(1-\alpha)}{\rho(\ell)}\right)\int_c^d p(\sigma(\ell,s),v)dv\right]ds$$

Where $\rho(\ell)$ is a positive real decreasing function with $\rho(\ell) \to 0$as$t \to \infty$.

$$\eta(\ell) = \int_{\ell_0}^t \psi^{-\frac{1}{\beta}}(s)ds,$$

$$\bar{\eta}(\ell) = \eta(\ell) + \frac{1}{\beta}\int_{\ell_1}^{\ell} \eta(u)\eta^\beta(\sigma(u,a)) \;\; E(u)du,$$

$$\mu(\ell) = \exp\left(-\beta\int_{\sigma(\ell)}^{\ell} \frac{du}{\bar{\eta}(u)\psi^{\frac{1}{\beta}}(u)}\right).$$

Next we provide some lemmas which are used to prove our results.

Lemma2.1.*[9]Let*$a > 0$*and*$0 < \alpha < 1$.*Then*

$$a^\alpha \leq \alpha a + (1-\alpha) \tag{2.1}$$

and equalityholds if $\alpha = 1$.

Lemma2.2. *[3] If* x*is a positive solution of*(E)*on* $[\ell_0,\infty)$,*then there exists*

$\ell_1 \in [\ell_0,\infty)$*suchthat*

$$z(\ell) > 0, z'(\ell) > 0, \left(\psi(\ell)\left(z'(\ell)\right)^\beta\right)' \leq 0 \tag{2.2}$$

on$[\ell_1,\infty)$.

Lemma2.3. *Let* x *be a positive solution of* (E). *The nthe function* z *satisfies*

$$\left(\psi(\ell)\left(z'(\ell)\right)^\beta\right)' \leq -E(\ell)z^\delta(\sigma(t,a)), \tag{2.3}$$

$$z(\ell) \geq \bar{\eta}(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell),\ if\ \delta = \beta, \tag{2.4}$$

and

$(\psi(\ell)\left(z'(\ell)\right)^{\beta})' \leq -E(\ell)\mu(\ell)z^{\beta}(\ell), if\ \delta = \beta$.(2.5)

Proof. Let x be a positive solution of (E) on $[\ell_0, \infty)$.Then there exists a $\ell_1 \geq \ell_0$such that $x(\tau(\ell, v)) > 0$and $x(\sigma(\ell, s)) > 0$for$\ell \geq \ell_1$,$v \in [c, d]$and$s \in [a, b]$.By Lemma2.2,

We obtain(2.2) holds. Also by the definition of$z(\ell)$,weget

$$\begin{aligned} \mathrm{x}(\ell) &= \mathrm{z}(\ell) - \int_c^d p(\ell, v)\, x^{\alpha}\left(\tau(\ell, v)\right) dv \\ &\geq z(\ell) - \int_c^d p(\ell, v)\, z^{\alpha}\left(\tau(\ell, v)\right) dv. \end{aligned} \tag{2.6}$$

Now using Lemma2.1 and $\rho(\ell)$ is decreasing and tending to zero, we have from (2.6)

that

$$\begin{aligned} \mathrm{x}(\ell) &\geq \left[\mathrm{z}(\ell) - \mathrm{z}^{\alpha}(\ell) \int_c^d p(\ell, v)\, dv\right] \\ &\geq z(\ell)\left[1 - \left(\alpha + \frac{(1-\alpha)}{\rho(\ell)}\right) \int_c^d p(\ell, v)\, dv\right] \end{aligned}$$

which,with(E),impliesthat

$$\left(\psi(\ell)\left(\mathrm{z}'(\ell)\right)^{\beta}\right)' \leq -\int_a^b q(\ell, s)\, z^{\delta}\left(\sigma(\ell, s)\right) \left[1 - \left(\alpha + \frac{(1-\alpha)}{\rho(\ell)}\right) \int_c^d p\left((\ell, s), v\right) dv\right]^{\delta} ds.$$

Since$z'(\ell) > 0$ and $\frac{\partial}{\partial s}\sigma(\ell, s) > 0$, we see that $\mathrm{z}\left(\sigma(\ell, s)\right) \geq z(\sigma(\ell, a))$ and so

$$\left(\psi(\ell)\left(z'(\ell)\right)^{\beta}\right)' \leq -E(\ell) z^{\delta}\left(\sigma(\ell, a)\right)$$

Which proves(2.3).

Employing the chain rule and simple calculation, easily we see that

$$\eta(\ell)\left(\psi(\ell)\left(z'(\ell)\right)^{\beta}\right)' = -\beta\left(\psi^{\frac{1}{\beta}}(\ell)z'(\ell)\right)^{\beta-1}\frac{d}{d\ell}\left(z(\ell)-\eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell)\right). \quad (2.7)$$

Combining (2.3)and(2.7),we get

$$\frac{d}{d\ell}\left(z(\ell)-\eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell)\right) \geq \frac{1}{\beta}\eta(\ell)(\psi^{\frac{1}{\beta}}(\ell)\mathrm{z}'(\ell))^{1-\beta}E(\ell)z^{\delta}\left(\sigma(\ell,a)\right).$$

Taking the integration, the previous in equality from ℓ_1 to ℓ, we have

$$\mathrm{z}(\ell) \geq \eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell) + \frac{1}{\beta}\int_{\ell_1}^{\ell}\eta(u)\,E(u)(\psi^{\frac{1}{\beta}}(\ell)\mathrm{z}'(\ell))^{1-\beta}z^{\delta}\left(\sigma(u,a)\right)du. \quad (2.8)$$

From the monotonicity of $\psi^{\frac{1}{\beta}}(\ell)z'(\ell)$, we have

$$\mathrm{z}(\ell) = \mathrm{z}(\ell_1) + \int_{\ell_1}^{\ell}\frac{\psi^{\frac{1}{\beta}}(u)z'(u)}{\psi^{\frac{1}{\beta}}(u)}du \geq \eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell).$$

Since $\psi^{\frac{1}{\beta}}(\ell)z'(\ell)$ is decreasing, (2.8) becomes

$$\mathrm{z}(\ell) \geq \eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell) + \frac{1}{\beta}\int_{\ell_1}^{\ell}\eta(u)\,E(u)(\psi^{\frac{1}{\beta}}(u)\mathrm{z}'(\mathrm{u}))^{1-\beta}\eta^{\delta}\left(\sigma(u,a)\right)\left(\psi^{\frac{1}{\beta}}\left(\sigma(u,a)\right)z'\left(\sigma(u,a)\right)\right)^{\delta}du$$

$$\geq \psi^{\frac{1}{\beta}}(\ell)z'(\ell)\left[\eta(\ell)+\frac{1}{\beta}\int_{\ell_1}^{\ell}\eta(u)\eta^{\delta}\left(\sigma(u,a)\right)E(u)\left(\psi^{\frac{1}{\beta}}(u)z'(u)\right)^{\delta-\beta}\right]du.$$

Set$\beta =$, thenweobtain (2.4).From (2.4),wehave

$$\frac{\mathrm{z}'(\ell)}{\mathrm{z}(\ell)} \leq \frac{1}{\overline{\eta}(\ell)\psi^{\frac{1}{\beta}}(\ell)}.$$

Taking integration the above inequality from $\sigma(\ell,a)$ to ℓ, yields

$$\frac{z(\sigma(\ell,a))}{z(\ell)} \geq \exp\left(-\int_{\sigma(\ell,a)}^{t} \frac{du}{\bar{\eta}(\ell)\psi^{\frac{1}{\beta}}(\ell)}\right)$$

Which with (2.3)for$\delta = \beta$gives

$$\frac{\left(\psi(\ell)\left(z'(\ell)\right)^{\beta}\right)'}{z^{\beta}(\ell)} \leq -E(\ell)\left(\frac{z\left(\sigma(\ell,a)\right)}{z(\ell)}\right)^{\beta} \leq -E(\ell)\mu(\ell)$$

Which proves (2.5). This completes the proof.

3. MainResults

In this section, we obtain the oscillation criteria of solutions of (E).

Theorem 3.1. *Let $\delta = \beta$, and there exists a positive decreasing function $\rho(\ell)$suchthat $\rho(\ell) \to 0$as $\ell \to \infty$and $E(\ell) > 0$for $\ell \geq \ell_1$. If the first order delay differential equation*

$$w'(\ell) + \bar{\eta}^{\beta}(\ell)\left(\sigma(\ell,a)\right)E(\ell)w\left(\sigma(\ell,a)\right) = 0 \qquad (3.1)$$

Is oscillatory ,then$(E)$is oscillatory.

*Proof.*Suppose (E) has a nonoscillatory solution $x$on $[\ell_0,\infty)$.With no loss of generality,weassumethat$x(\ell) > 0$,$x(\tau(\ell,v)) > 0$,and$x(\sigma(\ell,s)) > 0$for$\ell \geq \ell_1 \geq \ell_0, v \in [c,d]$ and $s \in [a,b]$.By Lemma 2.3,we obtain (2.3) and (2.4) hold.Using(2.3)and (2.4), one can see that $w(\ell) = \psi(\ell)\left(z'(\ell)\right)^{\beta}$is a positive solution of the first orderdelay differentialinequality

$$w'(\ell) + \bar{\eta}^{\beta}(\ell)(\sigma(\ell,a))E(\ell)w(\sigma(\ell,a)) \leq 0$$

But,in view of Theorem 1 of [18], the associate delay equation(3.1) also has a positive solution, which is a contradiction. This completes the proof.

Corollary3.2.*Let$\delta = \beta$ and there exists a positive decreasing function $\rho(\ell)$such that $\rho(\ell) \to 0$as$\ell \to \infty$and$E(\ell) > 0$forall$\ell \geq \ell_1$.If*

$$\lim_{\ell\to\infty} sup \int_{\sigma(\ell,a)}^{t} \bar{\eta}^{\beta}\big(\sigma(u,a)\big)\, E(u)du > 1, \frac{\partial}{\partial \ell}\sigma(\ell,s) \geq 0 \qquad (3.2)$$

or

$$\lim_{\ell\to\infty} inf \int_{\sigma(\ell,a)}^{t} \bar{\eta}^{\beta}\big(\sigma(u,a)\big)\, E(u)du > \frac{1}{e}, \qquad (3.3)$$

Then (E) *is oscillatory.*

Proof. It is familiar that (3.2) or (3.3) assures oscillation of (3.1) (by Theorem 2.1.1of[8]).Now the outcomes follow from Theorem 3.1. The proof is now complete.

Theorem3.3.*Let there exists a positive decreasing function*$\rho(\ell)$*such that*$\rho(\ell) \to 0$ *as*$\ell \to \infty$*and*$E(\ell) > 0$*for all*$\ell \geq \ell_1$*.If the first order delay differential equation*

$$w'(\ell) + E(\ell)\eta^{\delta}\big(\sigma(\ell,a)\big)w^{\frac{\delta}{\beta}}\big(\sigma(\ell,a)\big) = 0 \qquad (3.4)$$

Is oscillatory, then (E)*is oscillatory.*

Proof. Assume the contrary that (E)has a positive solution $x$on $[\ell_0,\infty)$. With no loss of generality, we assume that there exists a $\ell_1 \geq \ell_0$such that$x(\ell) > 0, x(\tau(\ell,v)) > 0$,and$x(\sigma(\ell,s)) > 0for\ell \geq \ell_1 \geq \ell_0, v \in [c,d]ands \in [a,b]$.Since$\psi^{\frac{1}{\beta}}(\ell)z'(\ell)$is decreasing,we have

$$\mathrm{z}(\ell) = \mathrm{z}(\ell_1) + \int_{\ell_1}^{\ell} \frac{z'(u)}{\psi^{\frac{1}{\beta}}(u)}du \geq \eta(\ell)\psi^{\frac{1}{\beta}}(\ell)z'(\ell). \qquad (3.5)$$

Using(3.5)and(2.3),we obtain $w(\ell) = \psi(\ell)\big(z'(\ell)\big)$is a positive solution of the first order delay differential inequality

$$\mathrm{w}'(\ell) + \mathrm{E}(\ell)\eta^{\delta}\big(\sigma(\ell,a)\big)w^{\frac{\delta}{\beta}}\big(\sigma(\ell,a)\big) \leq 0.$$

In view of Theorem 2.1 of [17], the related delay equation (3.4) also has a positive solution, we get a contradiction. The proof is complete.

Corollary3.4.*Let*$\delta < \beta$*andthereexistsapositivedecreasingcontinuousfunction*$\rho(\ell)$*such that* $\rho(\ell) \to 0$ *as* $\ell \to \infty$ *and* $E(\ell) > 0$ *for all* $\ell \geq \ell_1$*. If*

$$\int_{\ell_0}^{\ell} E(s)\,\eta^{\delta}\big(\sigma(s,a)\big)ds = \infty, \qquad (3.6)$$

then(E)isoscillatory

Proof. It is well-known that the inequality (3.5) ensures oscillation of(3.4) (seeTheorem 2 of [10]).Now the conclusion follows from Theorem 3.3.The proof iscompleted.

Corollary3.5.*Let$\delta > \beta$andthereexistsapositivedecreasingcontinuousfunction $\rho(\ell)$suchthat$\rho(\ell) \to 0$as$\ell \to \infty$and$E(\ell) > 0$forall$\ell \geq \ell_1$.If$\sigma(\ell, a) = \ell - k$, where$k > 0$isaconstant,andthereexistsλsuchthat$\lambda > \frac{1}{k}\ln\frac{\delta}{\beta}$and*

$$\lim_{\ell\to\infty}\inf\left[E(\ell)\eta^{\delta}\big(\sigma(\ell,a)\big)\exp\left(-e^{\lambda\ell}\right)\right] > 0, \quad (3.7)$$

Then (E) is oscillatory.

Proof. It is well-known that the inequality (3.7) ensures oscillation of(3.4) (see Theorem 3 of [19]).Now the conclusion follows from Theorem 3.3.The proof iscompleted. □

Corollary3.6.*Let$\delta > \beta$ and there exists a positive decreasing continuous function $\rho(\ell)$ such that $\rho(\ell) \to 0$as$\ell \to \infty$and$E(\ell) > 0$forall$\ell \geq \ell_1$.If$\sigma(\ell, a) = \theta\ell$, $\theta \in (0,1)$and there exists $\mu > -\frac{\ln\frac{\delta}{\beta}}{\ln\theta}$such that*

$$\lim_{\ell\to\infty}\inf\left[E(\ell)\eta^{\delta}\big(\sigma(\ell,a)\big)\exp(-\ell^{\mu})\right] > 0, \quad (3.8)$$

Then (E) is oscillatory.

Proof. It is well-known that the inequality (3.8) ensure the oscillation of (3.4) (see Theorem 4 of[19]).Now the conclusion follows from Theorem 3.3. The proof is now complete.

4. Examples

Here,we provide two illustrations to show the importance and novelty of our results.

Example4.1.*Considerthe differential equation with distributed deviating arguments of the form*

$$\left(x(\ell) + \frac{1}{\ell}x^{\frac{1}{3}}(\ell-2)\right)'' + \int_1^2 \frac{\ell+s}{\ell}x\left(\frac{\ell}{s}\right)ds = 0, \ell \geq 1. \quad (4.1)$$

Here $\psi(\ell) = 1, p(\ell, s) = \frac{1}{\ell}, \tau(\ell, s) = \ell - 2, q(\ell, s) = \frac{\ell+s}{\ell}, \sigma(\ell, s) = \frac{\ell s}{\ell}, \alpha = \frac{1}{3}, \beta = \delta = 1$. A simple calculation shows that $\eta(\ell) = \ell - 1$ and the conditions $(H_1) - (H_3)$ are satisfied. Further by choosing $\rho(\ell) = \frac{1}{\ell}$, $E(\ell) \approx \frac{1}{2} - \frac{M}{\ell}$, $M > 0, \bar{\eta}(\ell) \approx \frac{\ell^3}{12}$. Now condition (3.2) becomes

$$\lim_{\ell\to\infty}\sup\int_{\frac{\ell}{2}}^{\ell}\frac{u^3}{96}\left(\frac{1}{2}-\frac{M}{u}\right)du=\infty>1, and\ \frac{\partial}{\partial\ell}\sigma(\ell,s)=\frac{1}{2}>0$$

That is, condition (3.2) is satisfied. The condition(3.3)is also holds.Hence by Corollary 3.2, equation (4.1) is oscillatory.

Example4.2. *Consider the second-order differential equation with distributed deviating arguments of the form*

$$\left(x(\ell)+\int_1^2\frac{1}{4\ell}x^{\frac{1}{3}}\left(\frac{\ell s}{3}\right)\right)''+\int_0^1\ell s\,x^{\frac{1}{3}}\left(\frac{ts}{2}\right)ds=0,\ell\geq 1. \qquad (4.2)$$

Here $\psi(\ell)=1, \mathrm{p}(\ell,\mathrm{s})=\frac{1}{4\ell}, \tau(\ell,s)=\frac{\ell s}{3}, q(\ell,s)=\ell s, \sigma(\ell,s)=\frac{ts}{2}, \alpha=\frac{1}{3}, \beta=1, \delta=\frac{1}{3}, c=$ 1 and $\mathrm{d}=2$.It is easy to see that conditions$(H_1)-(H_3)$hold.A simple

Computation shows that $\eta(\ell)=\ell-1$,andletting$\rho(\ell)=\frac{1}{\ell}$, *we have*$\mathrm{E}(\ell)\approx\frac{\ell}{6}-\frac{1}{6}$. The condition (3.5) becomes

$$\int_1^\infty\left(\frac{\ell}{6}-\frac{1}{6}\right)\left(\frac{\ell}{6}-1\right)^{\frac{1}{3}}d\ell=\infty$$

Hence by Corollary 3.4, and (4.2) is oscillatory.

5. CONCLUSION

In this paper, we have used comparison method and integral averaging technique to get new conditions for the oscillation of all solutions of (*E*). Our new results extend and complement to many known results reported in the literature for second-order neutral differential equations with or without distributed deviating arguments.

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