Synchronization for BAM Complex-Valued Neural Networks with both Discrete and Distributed Successive Time-Varying Delays

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Abstract- This paper focus on the globally asymptotically synchronization for BAMCVNNs with both discrete and distributed successive time-varying delays. By constructing a proper Lyapunov–Krasovskii functional and using matrix inequality technique and reciprocally convex technique. Several delay-dependent conditions are presented in the form linear matrix inequalities effortlessly, which may be solved by means of the use of Matlab LMI toolbox. An illustrative example is given to show the feasibility of our theoretical results.

Keywords – Complex-valued neural networks, Successive time-varying delays, Synchronization, Linear matrix inequality.

I. INTRODUCTION

In the past few decades, neural networks (NNs) have been an important information processing system and play a crucial rule in the biological information and engineering field. In current years, there has been a large range of studies dedicated to NNs consisting of automatic control, medical image analysis, biological information process, artificial intelligence, signal processing and associative memory [1]-[6]. Moreover, real valued neural networks (RVNNs) have had great success in many areas, they likely perform worse on certain physical related application such as affine transformation. As an extension of RVNNs, complex value neural networks (CVNNs) are appropriate to solve such problems. CVNNs have complex valued states, complex-valued connection weights as well as complex-valued activation functions are very complicated. Due to the complex number characteristic, CVNNs can be used for many physical problems such as ultrasonic waves, electromagnetic waves, quantum waves, light and so on. In addition, CVNN can help you solve simple problems that

RVNNs cannot solve. For example, CVNNs can simply deal with the well-known XOR problem that cannot be solved with a single real-valued neuron [7]. As a result, many researchers have joined efforts to study CVNNs, particularly related dynamic analysis. Until now, many substantial results have been proposed in this area [8]-[12].

The research of associative memory network is likewise an crucial branch of NNs, and the bidirectional associative reminiscence (BAM) NNs proposed through Kosko have the widest packages in all styles of associative memory network models [13,14]. BAMNNs are double-layer bidirectional networks, wherein the neurons in a single layer are completely linked with the neurons within the other layer, and there are no connections among neurons inside the identical layer. When the enter signal is introduced to 1 layer, the output signal may be received from the alternative one.

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Moreover, the initial version can impact any layer in networks to motive bidirectional transmission of information. In addition, the study on time-delay system extensively exists inside the actual systems which include biomedical, engineering, physical and chemical. They can motive unwanted dynamical behaviors including instability and chaos. In widespread, the time-delay system may be categorized into kinds together with continuous case and discrete case. However, as compared to physics systems in continuous case and discrete case sensor systems have a robust background in digital network packages network-based manage is properly identified as an established for instance. In the above factor of delayed BAMCVNNs some big results had been published in [15]-[18].

The synchronization of CVNNs has been worried via numerous scholars and plenty of conclusions were drawn. In recent year, growing attention has been paid for the worldwide asymptotically synchronization problem for various kinds of CVNNs with time-varying delays and lots of effects have been suggested [19] – [22]. In [19], the authors investigated the problem of master–slave system realizes synchronization of uncertain CVNNs with discontinuous activation functions. A class of Cohen–Grossberg CVNNs the synchronization problem by using Lyapunov-functionals and adaptive control method is investigated [20]. In [21], the dynamical problem of memristor-based CVNNs with time delays and by utilizing the inequality technique is considered. Recently, the problem of finite-time synchronization for CVNNs with time-varying delay is discussed in [22].

In a networked control system, indicators transmitting from one point to some other skip thru a few network nodes, which can also cause a while delays during network propagation. In the process of network propagation, the traits of these time delays aren't the identical, so they can't be mixed to take a look at [23–26]. Therefore, the model of NNs with two additive time-varying delay components is proposed due to realistic application historical past. Recently, via introducing a reciprocally convex inequality, the state estimation of NNs with additive time-varying delay studied [27]. Until now, the synchronization and the stableness of CVNNs with single delay have been investigated [28]-[31]. Very recently, in [32] the authors proposed the synchronization problem of CVNNs with two additive time-varying delays. However, to the best of authors knowledge, so far, no result on the synchronization for BAMCVNNs with successive time-varying delays have been reported.

Inspired the above discussions, this paper studies the globally asymptotically synchronization for BAMCVNNs with both discrete and distributed successive time-varying delays. The main contributions are as follows:

- The synchronization analysis results for BAMCVNNs with discrete and distributed successive time-varying delays are proposed for the first time.
- A new Lyapunov–Krasovskii function built on this paper is more complete by way of inclusive of full facts of discrete and distributed successive time-varying delays.
- The reciprocally convex technique in complex-valued discipline is carried out to lower the conservatism of the received standards.
- The conditions in our fundamental results can be converted into linear matrix inequalities effortlessly, which may be solved by means of the use of Matlab LMI toolbox.

The contributions of the above strategies are demonstrated via a numerical example.

II. MODEL DESCRIPTION AND PRELIMINARIES

Consider the BAM Complex-Valued Neural Networks with discrete and distributed additive time-varying delays as follows:

$$\dot{x}(t) = -Cx(t) + W_1 f(x(t)) + W_2 f(x(t - \sigma_1(t) - \sigma_2(t))) + W_3\int_{t-\delta_1(t)-\delta_2(t)}^{t} f(x(s))ds + I$$

$$\dot{y}(t) = -Dy(t) + Z_1 g(y(t)) + Z_2 g(y(t - \tau_1(t) - \tau_2(t))) + Z_3\int_{t-h_1(t)-h_2(t)}^{t} g(y(s))ds + J \qquad (1)$$

where $x(t) = [x_1(t),x_2(t), . . . ,x_n(t)]^T \epsilon C^n$, $y(t) = [y_1(t),y_2(t), . . . ,y_n(t)]^T \epsilon C^n$ stands for the state vector of the neurons; $C = diag\{c_1,c_2, . . . ,c_n\} \epsilon R^{nxn}$, $D = diag\{d_1,d_2,. . .,d_n\} \epsilon R^{nxn}$ and $c_k>0$, $d_k>0$, $k = 1,2, . . . ,n$; $W_1 \epsilon C^{nxn}$, $W_2 \epsilon C^{nxn}$, $W_3 \epsilon C^{nxn}$, $Z_1 \epsilon C^{nxn}$, $Z_2 \epsilon C^{nxn}$ and $Z_3 \epsilon C^{nxn}$ represent the connection coefficient matrices; $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), . . . , f_n(x_n(t))]^T \epsilon C^n$, $g(y(t)) = [g_1(y_1(t)), g_2(y_2(t)), . . . , g_n(y_n(t))]^T \epsilon C^n$ shows the activation function of neurons; $I \epsilon C^n$, $J \epsilon C^n$ are the external input; $\sigma_1(t)$, $\sigma_2(t)$, $\tau_1(t)$ and $\tau_2(t)$ denote the discrete time-varying delays; $\delta_1(t)$, $\delta_2(t)$, $h_1(t)$ and $h_2(t)$ denote the distributed time-varying delays; These kinds of time-varying delays meet the following conditions,

$$0 \le \sigma_i(t) \le \sigma_i,\ 0 \le \tau_i(t) \le \tau_i$$
$$0 \le \delta_i(t) \le \delta_i,\ 0 \le h_i(t) \le h_i,\ i = 1,2. \qquad (2)$$

where σ_1, σ_2, τ_1, τ_2, δ_1, δ_2, h_1 and h_2 are constants. Let

$$\sigma(t) = \sigma_1(t) + \sigma_2(t),\ \tau(t) = \tau_1(t) + \tau_2(t),$$
$$\delta(t) = \delta_1(t) + \delta_2(t),\ h(t) = h_1(t) + h_2(t) \text{ and}$$
$$\sigma = \sigma_1 + \sigma_2,\ \tau = \tau_1 + \tau_2,\ \delta = \delta_1 + \delta_2,\ h = h_1 + h_2.$$

The initial value of BAM Complex-Valued Neural Networks (1) is given to be

$$x(s) = \Psi(s)$$
$$y(s) = \varphi(s),\ s \epsilon [-\alpha,0]. \qquad (3)$$

Where $\Psi \epsilon C^n$ and $\varphi \epsilon C^n$ are continuous in $[-\alpha,0]$ with $\alpha = max\{\sigma, \tau, \delta, h\}$.

For convenience, we denote model (1) as the master system and consider the corresponding slave system as:

$\dot{u}(t) = -Cu(t) + W_1 f(u(t)) + W_2 f(u(t - \sigma_1(t) - \sigma_2(t))) + W_3\int_{t-\delta_1(t)-\delta_2(t)}^{t} f(u(s))ds + I + E(t),$

$\dot{v}(t) = -Dv(t) + Z_1 g(v(t)) + Z_2 g(v(t - \tau_1(t) - \tau_2(t))) + Z_3\int_{t-h_1(t)-h_2(t)}^{t} g(v(s))ds + J + F(t)$

(4)

with initial value $u(s) = \rho(s)$ and $v(s) = \zeta(s)$, $s \in [-\alpha,0]$ and $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in C^n$, $v(t) = [v_1(t), v_2(t), \ldots, v_n(t)]^T \in C^n$ for $t \geq 0$ and E(t), F(t) are the controller. Let $\bar{\beta}(t) = u(t) – x(t)$, $\bar{\gamma}(t) = v(t) – y(t)$ be the error state and design the controller to be

$$E(t) = K_1\bar{\beta}(t) + K_2\bar{\beta}(t – \sigma(t)) + K_3\bar{\beta}(t – \delta(t)),$$
$$F(t) = L_1\bar{\gamma}(t) + L_2\bar{\gamma}(t – \tau(t)) + L_3\bar{\gamma}(t – h(t)). \quad (5)$$

where $K_1, K_2, K_3, L_1, L_2, L_3 \in C^{n\times n}$ are the gain matrices. Then, based on (1), (4) and (5), the synchronization error of BAM Complex-Valued Neural Networks system is derived as follows:

$\dot{\bar{\beta}}(t) = (-C + K_1)\bar{\beta}(t) + K_2\bar{\beta}(t - \sigma(t)) + K_3\bar{\beta}(t - \delta(t)) + W_1 f(\bar{\beta}(t)) + W_2 f(\bar{\beta}(t - \sigma(t))) + W_3\int_{t-\delta(t)}^{t} f(\bar{\beta}(s))ds,$

$\dot{\bar{\gamma}}(t) = (-D + L_1)\bar{\gamma}(t) + L_2\bar{\gamma}(t - \tau(t)) + L_3\bar{\gamma}(t - h(t)) + Z_1 g(\bar{\gamma}(t)) + Z_2 g(\bar{\gamma}(t - \tau(t))) + Z_3\int_{t-h(t)}^{t} g(\bar{\gamma}(s))ds$ (6)

where $f(\bar{\beta}(t)) = f(u(t)) – f(x(t))$, $g(\bar{\gamma}(t)) = g(v(t)) – g(y(t))$, the initial values of error system (6) is considered as

$$\bar{\beta}(s) = \eta(s) – \Psi(s),$$
$$\bar{\gamma}(s) = \eta(s) – \varphi(s),\ s \in [-\alpha, 0]$$

To establish the main result, the following assumption should be satisfied.

(A). For $i \in \{1,2...n\}$, $j \in \{1,2...n\}$, the activation function $f_i(\cdot)$ and $g_j(\cdot)$ are continuous. Further, there exists a positive matrix $M = diag\{m_1, m_2, \ldots, m_n\}$ and $N = diag\{n_1, n_2, \ldots, n_n\}$ such that

$$|f_i(\alpha_1) – f_i(\alpha_2)| \leq m_i |\alpha_1 - \alpha_2|$$
$$|g_j(\beta_1) – g_j(\beta_2)| \leq n_j |\beta_1 - \beta_2|.$$

Definition 1

System (1) and system (4) is said to be globally asymptotically synchronized, if system (6) is globally asymptotically stable.

Lemma 1 [33]

Suppose $X \in C^{m\times m}$ as a Hermitian matrix and $X>0$, $0< \epsilon(t) < \epsilon$ is a scalar function while $\xi(\cdot):[0,\epsilon]\longrightarrow C^m$ is a vector function, one has

$$\left[\int_{t-\epsilon(t)}^{t}\xi(s)ds\right]^{*}\times\left[\int_{t-\epsilon(t)}^{t}\xi(s)ds\right]\leq \epsilon(t)\int_{t-\epsilon(t)}^{t}\xi^{*}(s)\times\xi(s)ds.$$

Lemma 2 [34]

For arbitrary vector $\eta\in C^{m}$, scalar $\beta\in(0,1)$, a positive define matrix $Q\in C^{nxn}$ and matrices $B_1,B_2\in C^{nxm}$, the function $\Psi(\beta,Q)$ is defined as

$$\Psi(\beta,Q)=\frac{1}{\beta}\eta^{*}B_1^{*}QB_1\eta+\frac{1}{1-\beta}\eta^{*}B_2^{*}QB_2\eta$$

If the matrix $Y\in C^{nxn}$ satisfies $\begin{bmatrix} Q & Y \\ Y^{*} & Q \end{bmatrix}>0$,then

$$\min_{\beta\in(0,1)}\Psi(\beta,Q)\geq\begin{bmatrix} B_1\eta \\ B_2\eta \end{bmatrix}^{*}\begin{bmatrix} Q & Y \\ Y^{*} & Q \end{bmatrix}\begin{bmatrix} B_1\eta \\ B_2\eta \end{bmatrix}.$$

III. MAIN TITLE

In this section, we propose synchronization criterion for the BAMCVNNs with successive time-delay components.

Theorem 1 Let the assumption (A) hold. If there are fourteen Hermite matrices $S_i\in C^{nxn}$ and $S_i>0(i=1,2,3,\ldots\ldots,14)$, Eight positive diagonal matrices $R_j, T_j\in R^{nxn}(j=1,2,3,4)$, matrices X_{11}, X_{12}, X_{13}, X_{22}, X_{23}, X_{33}, Y_{11}, Y_{12}, Y_{13}, Y_{22}, Y_{23}, Y_{33}, G_{11}, G_{12}, G_{22}, H_{11}, H_{12}, H_{22}, P_{11}, P_{12}, P_{22}, Q_{11}, Q_{12}, Q_{22}, U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, E_1, E_2, E_3, F_1, F_2, F_3, $A,B\in C^{nxn}$ such that the following LMI'S hold;

$$X=\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix}>0, \quad (7)$$

$$Y=\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix}>0, \quad (8)$$

$$G=\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}>0, \quad (9)$$

$$H=\begin{bmatrix} H_{11} & H_{12} \\ * & H_{22} \end{bmatrix}>0, \quad (10)$$

$$P=\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}>0, \quad (11)$$

$$Q=\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}>0, \quad (12)$$

$$\begin{bmatrix} S_5 & U_1 \\ * & S_5 \end{bmatrix}>0, \quad (13)$$

$$\begin{bmatrix} S_6 & U_2 \\ * & S_6 \end{bmatrix}>0, \quad (14)$$

$$\begin{bmatrix} S_7 & U_3 \\ * & S_7 \end{bmatrix}>0, \quad (15)$$

$$\begin{bmatrix} S_8 & U_4 \\ * & S_8 \end{bmatrix}>0, \quad (16)$$

$$\begin{bmatrix} S_{11} & U_5 \\ * & S_{11} \end{bmatrix} > 0, \quad (17)$$

$$\begin{bmatrix} S_{12} & U_6 \\ * & S_{12} \end{bmatrix} > 0, \quad (18)$$

$$\begin{bmatrix} S_{13} & U_7 \\ * & S_{13} \end{bmatrix} > 0, \quad (19)$$

$$\begin{bmatrix} S_{14} & U_8 \\ * & S_{14} \end{bmatrix} > 0, \quad (20)$$

$$\theta = (\theta_{ij})_{34\times 34} < 0, \quad (21)$$

where

$\theta_{1,1} = \sigma_1^2 S_5 + \sigma^2 S_6 + \delta_1^2 S_{11} + \delta^2 S_{12} - A - A^*$,

$\theta_{1,2} = X_{11} - AC + E_1 - A^*$, $\theta_{1,6} = E_2$, $\theta_{1,7} = X_{12}$,

$\theta_{1,8} = X_{13}$, $\theta_{1,9} = AW_1$, $\theta_{1,15} = E_3$, $\theta_{1,16} = AW_2$, $\theta_{1,17} = AW_3$,

$\theta_{2,2} = X_{12} + X_{13} + X_{21} + X_{31} + G_{11} + \sigma_1^2 S_1 + \sigma^2 S_2 - S_5 - S_6 - S_{11} - S_{12}$
$+ MR_1M - AC - CA^* + E_1 + E_1^*$,

$\theta_{2,3} = -X_{12} + U_1^*$, $\theta_{2,4} = -X_{13} + U_2^*$, $\theta_{2,5} = S_5 - U_1^*$,

$\theta_{2,6} = S_6 - U_2^* + E_2$, $\theta_{2,7} = X_{22} + X_{32}$, $\theta_{2,8} = X_{23} + X_{33}$,

$\theta_{2,9} = G_{12} + AW_1$, $\theta_{2,12} = U_5^*$, $\theta_{2,13} = S_{11} - U_5^*$,

$\theta_{2,14} = U_6^*$, $\theta_{2,15} = S_{12} - U_6^* + E_3$, $\theta_{2,16} = AW_2$,

$\theta_{2,17} = AW_3$, $\theta_{3,3} = H_{11} - G_{11} - S_5 + MR_2M$, $\theta_{3,5} = S_5 - U_1$,

$\theta_{3,7} = -X_{33}$, $\theta_{3,8} = -X_{23}$, $\theta_{3,10} = H_{12} - G_{12}$, $\theta_{4,4} = -H_{11} - S_6 + MR_3M$,

$\theta_{4,6} = S_6 - U_2$, $\theta_{4,7} = -X_{32}$, $\theta_{4,8} = -X_{33}$, $\theta_{4,11} = -H_{12}$,

$\theta_{5,5} = -2S_5 + U_1 + U_1^*$, $\theta_{6,6} = -2S_6 + U_2 + U_2^* + MR_4M$,

$\theta_{7,7} = -S_1$, $\theta_{8,8} = -S_2$, $\theta_{9,9} = G_{22} + \delta^2 S_9 - R_1$, $\theta_{10,10} = H_{22} - G_{22} - R_2$,

$\theta_{11,11} = -H_{22} - R_3$, $\theta_{12,12} = -S_{11}$, $\theta_{12,13} = S_{11} - U_5$,

$\theta_{13,13} = -2S_{11} + U_5 + U_5^*$, $\theta_{14,14} = -S_{12}$, $\theta_{14,15} = S_{12} - U_6$,

$\theta_{15,15} = -2S_{12} + U_6 + U_6^*$, $\theta_{16,16} = -R_4$,

$\theta_{17,17} = -S_9$, $\theta_{18,18} = \tau_1^2 S_7 + \tau^2 S_8 + h_1^2 S_{13} + h^2 S_{14} - B - B^*$,

$\theta_{18,19} = Y_{11} - BD + F_1 - B^*$, $\theta_{18,23} = F_2$, $\theta_{18,24} = Y_{12}$,

$\theta_{18,25} = Y_{13}$, $\theta_{18,26} = BZ_1$, $\theta_{18,32} = F_3$, $\theta_{18,33} = BZ_2$, $\theta_{18,34} = BZ_3$,

$\theta_{19,19} = Y_{12} + Y_{13} + Y_{21} + Y_{31} + P_{11} + \tau_1^2 S_3 + \tau^2 S_4 - S_7 - S_8 - S_{13} - S_{14}$
$+ NT_1N - BD - D^*B + F_1 + F_1^*$,

$\theta_{19,20} = -Y_{12} + U_3^*$, $\theta_{19,21} = -Y_{13} + U_4^*$, $\theta_{19,22} = S_7 - U_3^*$,

$\theta_{19,23} = S_8 - U_4^* + F_2$, $\theta_{19,24} = Y_{22} + Y_{32}$, $\theta_{19,25} = Y_{23} + Y_{33}$,

$\theta_{19,26} = P_{12} + BZ_1$, $\theta_{19,29} = U_7^*$, $\theta_{19,30} = S_{13} - U_7^*$, $\theta_{19,31} = U_8^*$,

$\theta_{19,32} = S_{14} - U_8^* + F_3$, $\theta_{19,33} = BZ_2$, $\theta_{19,34} = BZ_3$,

$\theta_{20,20} = Q_{11} - P_{11} - S_7 + NT_2N$, $\theta_{20,22} = S_5 - U_1$, $\theta_{20,24} = -Y_{33}$,

$\theta_{20,25} = -Y_{23}$, $\theta_{20,27} = Q_{12} - P_{12}$, $\theta_{21,21} = -Q_{11} - S_8 + NT_3N$,

$\theta_{21,23} = S_8 - U_4$, $\theta_{21,24} = -Y_{32}$, $\theta_{21,25} = -Y_{33}$, $\theta_{21,28} = -Q_{12}$,

$\theta_{22,22} = -2S_7 + U_3 + U_3^*$, $\theta_{23,23} = -2S_8 + U_4 + U_4^* + NT_4N$,

$\theta_{24,24} = -S_3$, $\theta_{25,25} = -S_4$, $\theta_{26,26} = P_{22} + h^2 S_{10} - T_1$,

$\theta_{27,27} = Q_{22} - P_{22} - T_2$, $\theta_{28,28} = -Q_{22} - T_3$, $\theta_{29,29} = -S_{13}$,

$\theta_{29,30} = S_{13} - U_7$, $\theta_{30,30} = -2S_{13} + U_7 + U_7^*$, $\theta_{31,31} = -S_{14}$,
$\theta_{31,32} = S_{14} - U_8$, $\theta_{32,32} = -2S_{14} + U_8 + U_8^*$, $\theta_{33,33} = -T_4$, $\theta_{34,34} = -S_{10}$

and $\theta_{ij} = 0$ for the other values of θ, then the global synchronization of slave model (4) and master system (1) is achieved. Addition, the gain matrices K_i and L_i (i = 1,2,3) are designed to

$$K_i = A^{-1}E_i \quad (22)$$

$$L_i = B^{-1}F_i \quad (23)$$

Proof:

Let

$\eta_1(t) = (\bar{\beta}^*(t), \int_{t-\sigma_1}^{t} \bar{\beta}^*(s)ds, \int_{t-\sigma}^{t} \bar{\beta}^*(s)ds)^*$,

$\eta_2(t) = (\bar{\gamma}^*(t), \int_{t-\tau_1}^{t} \bar{\gamma}^*(s)ds, \int_{t-\tau}^{t} \bar{\gamma}^*(s)ds)^*$,

$\eta_3(t) = (\bar{\beta}^*(t), f^*(\bar{\beta}(t)))^*$,

$\eta_4(t) = (\bar{\gamma}^*(t), g^*(\bar{\gamma}(t)))^*$.

Consider the following Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^{6} V_i(t), \quad (i = 1,2,3,4,5,6) \quad (24)$$

Where

$V_1(t) = \eta_1^*(t)X\eta_1(t) + \eta_2^*(t)Y\eta_2(t)$,

$V_2(t) = \int_{t-\sigma_1}^{t} \eta_3^*(s)G\eta_3(s)ds + \int_{t-\sigma}^{t-\sigma_1} \eta_3^*(s)H\eta_3(s)ds + \int_{t-\tau_1}^{t} \eta_4^*(t)P\eta_4(s)ds$
$+ \int_{t-\tau}^{t-\tau_1} \eta_4^*(s)Q\eta_4(s)ds$,

$V_3(t) = \sigma_1\int_{-\sigma_1}^{0}\int_{t+\theta}^{t} \bar{\beta}^*(s)S_1\bar{\beta}(s)dsd\theta + \sigma\int_{-\sigma}^{0}\int_{t+\theta}^{t} \bar{\beta}^*(s)S_2\bar{\beta}(s)dsd\theta$
$+ \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \bar{\gamma}^*(s)S_3\bar{\gamma}(s)dsd\theta + \tau\int_{-\sigma}^{0}\int_{t+\theta}^{t} \bar{\gamma}^*(s)S_4\bar{\gamma}(s)dsd\theta$,

$V_4(t) = \sigma_1\int_{-\sigma_1}^{0}\int_{t+\theta}^{t} \dot{\bar{\beta}}^*(s)S_5\dot{\bar{\beta}}(s)dsd\theta + \sigma\int_{-\sigma}^{0}\int_{t+\theta}^{t} \dot{\bar{\beta}}^*(s)S_6\dot{\bar{\beta}}(s)dsd\theta$
$+ \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{\bar{\gamma}}^*(s)S_7\dot{\bar{\gamma}}(s)dsd\theta + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\bar{\gamma}}^*(s)S_8\dot{\bar{\gamma}}(s)dsd\theta$,

$V_5(t) = \delta\int_{-\delta}^{0}\int_{t+\xi}^{t} f^*(\bar{\beta}(s))S_9 f(\bar{\beta}(s))dsd\xi + h\int_{-h}^{0}\int_{t+\xi}^{t} g^*(\bar{\gamma}(s))S_{10}\, g(\bar{\gamma}(s))dsd\theta$,

$V_6(t) = \delta_1\int_{-\delta_1}^{0}\int_{t+\theta}^{t} \dot{\bar{\beta}}^*(s)S_{11}\dot{\bar{\beta}}(s)dsd\theta + = \delta\int_{-\delta}^{0}\int_{t+\theta}^{t} \dot{\bar{\beta}}^*(s)S_{12}\dot{\bar{\beta}}(s)dsd\theta$
$+ h_1\int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{\bar{\gamma}}^*(s)S_{13}\dot{\bar{\gamma}}(s)dsd\theta + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{\gamma}}^*(s)S_{13}\dot{\bar{\gamma}}(s)dsd\theta$.

Calculating the derivative of $V_1(t)$ along BAM CVNNs system (6), we obtain

$\dot{V}_1(t) = \eta_1^*(t)X\dot{\eta}_1(t) + \dot{\eta}_1^*(t)X\eta_1(t) + \eta_2^*(t)Y\dot{\eta}_2(t) + \dot{\eta}_2^*(t)Y\eta_2(t)$

$$= \begin{bmatrix} \bar{\beta}(t) \\ \int_{t-\sigma_1}^{t} \bar{\beta}(s)ds \\ \int_{t-\sigma}^{t} \bar{\beta}(s)ds \end{bmatrix}^* \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \begin{bmatrix} \dot{\bar{\beta}}(t) \\ \bar{\beta}(t) - \bar{\beta}(t-\sigma_1) \\ \bar{\beta}(t) - \bar{\beta}(t-\sigma) \end{bmatrix}$$

$$+\begin{bmatrix}\dot{\bar{\beta}}(t)\\ \bar{\beta}(t)-\bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t)-\bar{\beta}(t-\sigma)\end{bmatrix}^* \begin{bmatrix}X_{11} & X_{12} & X_{13}\\ X_{21} & X_{22} & X_{23}\\ X_{31} & X_{32} & X_{33}\end{bmatrix}\begin{bmatrix}\bar{\beta}(t)\\ \int_{t-\sigma_1}^{t}\bar{\beta}(s)ds\\ \int_{t-\sigma}^{t}\bar{\beta}(s)ds\end{bmatrix}$$

$$+\begin{bmatrix}\bar{\gamma}(t)\\ \int_{t-\tau_1}^{t}\bar{\gamma}(s)ds\\ \int_{t-\tau}^{t}\bar{\gamma}(s)ds\end{bmatrix}^* \begin{bmatrix}Y_{11} & Y_{12} & Y_{13}\\ Y_{21} & Y_{22} & Y_{23}\\ Y_{31} & Y_{32} & Y_{33}\end{bmatrix}\begin{bmatrix}\dot{\bar{\gamma}}(t)\\ \bar{\gamma}(t)-\bar{\gamma}(t-\tau_1)\\ \bar{\gamma}(t)-\bar{\gamma}(t-\tau)\end{bmatrix}$$

$$+\begin{bmatrix}\dot{\bar{\gamma}}(t)\\ \bar{\gamma}(t)-\bar{\gamma}(t-\tau_1)\\ \bar{\gamma}(t)-\bar{\gamma}(t-\tau)\end{bmatrix}^* \begin{bmatrix}Y_{11} & Y_{12} & Y_{13}\\ Y_{21} & Y_{22} & Y_{23}\\ Y_{31} & Y_{32} & Y_{33}\end{bmatrix}\begin{bmatrix}\bar{\gamma}(t)\\ \int_{t-\tau_1}^{t}\bar{\gamma}(s)ds\\ \int_{t-\tau}^{t}\bar{\gamma}(s)ds\end{bmatrix}$$

$= \dot{\bar{\beta}}^*(t)X_{11}\bar{\beta}(t) + \dot{\bar{\beta}}^*(t)X_{12}\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds + \dot{\bar{\beta}}^*(t)X_{13}\int_{t-\sigma}^{t}\bar{\beta}(s)ds$
$+ \bar{\beta}^*(t)(X_{12} + X_{13} + X_{21} + X_{31})\bar{\beta}(t) - \bar{\beta}^*(t)X_{12}\bar{\beta}(t - \sigma_1) - \bar{\beta}^*(t)X_{13}\bar{\beta}(t - \sigma)$
$+ \bar{\beta}^*(t)(X_{22} + X_{32})\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds + \bar{\beta}^*(t)(X_{23} + X_{33})\int_{t-\sigma}^{t}\bar{\beta}(s)ds$
$- \bar{\beta}^*(t - \sigma_1)X_{22}\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds - \bar{\beta}^*(t - \sigma_1)X_{23}\int_{t-\sigma}^{t}\bar{\beta}(s)ds$
$- \bar{\beta}^*(t - \sigma)X_{33}\int_{t-\sigma}^{t}\bar{\beta}(s)ds + \bar{\beta}^*(t)X_{11}\dot{\bar{\beta}}(t) + (\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds)^*X_{21}\dot{\bar{\beta}}(t)$
$+ (\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds)^*X_{22}\bar{\beta}(t) - (\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds)^*X_{22}\bar{\beta}(t - \sigma_1)$
$+ (\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds)^*X_{23}\bar{\beta}(t) - (\int_{t-\sigma_1}^{t}\bar{\beta}(s)ds)^*X_{23}\bar{\beta}(t - \sigma) + (\int_{t-\sigma}^{t}\bar{\beta}(s)ds)^*X_{31}\dot{\bar{\beta}}(t)$
$+ (\int_{t-\sigma}^{t}\bar{\beta}(s)ds)^*X_{32}\bar{\beta}(t) - (\int_{t-\sigma}^{t}\bar{\beta}(s)ds)^*X_{32}\bar{\beta}(t - \sigma_1) + (\int_{t-\sigma}^{t}\bar{\beta}(s)ds)^*X_{33}\bar{\beta}(t)$
$- (\int_{t-\sigma}^{t}\bar{\beta}(s)ds)^*X_{33}\bar{\beta}(t - \sigma) - \bar{\beta}^*(t - \sigma)X_{31}\bar{\beta}(t) - \bar{\beta}^*(t - \sigma_1)X_{21}\bar{\beta}(t) + \dot{\bar{\gamma}}^*(t)Y_{11}\bar{\gamma}(t)$
$+ \dot{\bar{\gamma}}^*(t)Y_{12}\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds + \dot{\bar{\gamma}}^*(t)Y_{13}\int_{t-\tau}^{t}\bar{\gamma}(s)ds + \bar{\gamma}^*(Y_{12} + Y_{13} + Y_{21} + Y_{31})\bar{\gamma}(t)$
$- \bar{\gamma}^*(t)Y_{12}\bar{\gamma}(t - \tau_1) - \bar{\gamma}^*(t)Y_{13}\bar{\gamma}(t - \tau) + \bar{\gamma}^*(t)(Y_{22} + Y_{32})\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds$
$+\bar{\gamma}^*(t)(Y_{23} + Y_{33})^*\int_{t-\tau}^{t}\bar{\gamma}(s)ds - \bar{\gamma}(t - \tau_1)Y_{22}\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds$
$- \bar{\gamma}^*(t - \tau_1)Y_{23}\int_{t-\tau}^{t}\bar{\gamma}(s)ds - \bar{\gamma}^*(t- \tau)Y_{32}\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds + \bar{\gamma}^*(t - \tau)Y_{33}\int_{t-\tau}^{t}\bar{\gamma}(s)ds$
$+ \bar{\gamma}^*(t)Y_{11}\dot{\bar{\gamma}}(t) + (\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds)^*Y_{21}\dot{\bar{\gamma}}(t) + (\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds)^*Y_{22}\bar{\gamma}(t)$
$-(\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds)^*Y_{22}\bar{\gamma}(t - \tau_1) + (\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds)^*Y_{23}\bar{\gamma}(t) - (\int_{t-\tau_1}^{t}\bar{\gamma}(s)ds)^*Y_{23}\bar{\gamma}(t - \tau)$
$+ (\int_{t-\tau}^{t}\bar{\gamma}(s)ds)^*Y_{31}\dot{\bar{\gamma}}(t) + (\int_{t-\tau}^{t}\bar{\gamma}(s)ds)^*Y_{32}\bar{\gamma}(t) - (\int_{t-\tau}^{t}\bar{\gamma}(s)ds)^*Y_{32}\bar{\gamma}(t - \tau)$
$+(\int_{t-\tau}^{t}\bar{\gamma}(s)ds)^*Y_{33}\bar{\gamma}(t) - (\int_{t-\tau}^{t}\bar{\gamma}(s)ds)^*Y_{33}\bar{\gamma}(t - \tau).$
(25)

$\dot{V}_2(t) = \eta_3^*(t)G\eta_3(t) + \eta_3^*(t - \sigma_1)(H - G)\eta_3(t - \sigma_1) - \eta_3^*(t - \sigma)H\eta_3(t-\sigma)$
$+ \eta_4^*(t)P\eta_4(t) + \eta_4^*(t - \tau_1)(Q - P)\eta_4(t - \tau_1) - \eta_4^*(t - \tau)Q\eta_4(t - \tau)$

$\dot{V}_2(t) = \bar{\beta}^*(t) G_{11}\bar{\beta}(t) + \bar{\beta}(t)G_{12}f(\bar{\beta}(t)) + f^*(\bar{\beta}(t))G_{21}\bar{\beta}(t)$
$+f^*(\bar{\beta}(t))G_{22} f(\bar{\beta}(t)) + \bar{\beta}^*(t - \sigma_1)(H_{11} - G_{11})\bar{\beta}(t - \sigma_1)$

$$+ \bar{\beta}^*(t - \sigma_1)(H_{12} - G_{12})f(\bar{\beta}(t - \sigma_1)) + f^*(\bar{\beta}(t - \sigma_1))(H_{21} - G_{21})\bar{\beta}(t - \sigma_1)$$
$$+ f^*(\bar{\beta}(t - \sigma_1))(H_{22} - G_{22})f(\bar{\beta}(t - \sigma_1)) - \bar{\beta}^*(t - \sigma)H_{11}\bar{\beta}(t - \sigma)$$
$$- \bar{\beta}^*(t - \sigma)H_{12} f(\bar{\beta}(t - \sigma)) - f^*(\bar{\beta}(t - \sigma))H_{21}\bar{\beta}(t - \sigma)$$
$$- f^*(\bar{\beta}(t - \sigma))H_{22} f(\bar{\beta}(t - \sigma)) + \bar{\gamma}^*(t)P_{11}\bar{\gamma}(t) + \bar{\gamma}^*(t)P_{12}g(\bar{\gamma}(t))$$
$$+ g^*(\bar{\gamma}(t))P_{21}\bar{\gamma}(t) + g^*(\bar{\gamma}(t))P_{22}g(\bar{\gamma}(t)) + \bar{\gamma}^*(t - \tau_1)(Q_{11} - P_{11})\bar{\gamma}(t - \tau_1)$$
$$+ \bar{\gamma}^*(t - \tau_1)(Q_{12} - P_{12})g(\bar{\gamma}(t - \tau_1)) + g^*(\bar{\gamma}(t - \tau_1))(Q_{21} - P_{21})\bar{\gamma}(t - \tau_1)$$
$$+ g^*(\bar{\gamma}(t - \tau_1))(Q_{22} - P_{22})g(\bar{\gamma}(t - \tau_1)) - \bar{\gamma}^*(t - \tau)Q_{11}\bar{\gamma}(t - \tau)$$
$$- \bar{\gamma}^*(t - \tau)Q_{12}g(\bar{\gamma}(t - \tau)) - g^*(\bar{\gamma}(t - \tau))Q_{21}\bar{\gamma}(t - \tau)$$
$$- g^*(\bar{\gamma}(t - \tau))Q_{22}\, g(\bar{\gamma}(t - \tau)). \quad (26)$$

$$\dot{V}_3(t) \leq \bar{\beta}^*(t)(\sigma_1^2 S_1 + \sigma^2 S_2)\bar{\beta}(t) - \left(\int_{t-\sigma_1}^{t} \bar{\beta}(s)ds\right)^* S_1\left(\int_{t-\sigma_1}^{t} \bar{\beta}(s)ds\right)$$
$$- \left(\int_{t-\sigma}^{t} \bar{\beta}(s)ds\right)^* S_2\left(\int_{t-\sigma}^{t} \bar{\beta}(s)ds\right) + \bar{\gamma}^*(t)(\tau_1^2 S_3 + \tau^2 S_4)\bar{\gamma}(t)$$
$$- \left(\int_{t-\tau_1}^{t} \bar{\gamma}(s)ds\right)^* S_3\left(\int_{t-\tau_1}^{t} \bar{\gamma}(s)ds\right) - \left(\int_{t-\tau}^{t} \bar{\gamma}(s)ds\right)^* S_4\left(\int_{t-\tau}^{t} \bar{\gamma}(s)ds\right). \quad (27)$$

$$\dot{V}_4(t) = \dot{\bar{\beta}}^*(t)(\sigma_1^2 S_5 + \sigma^2 S_6)\dot{\bar{\beta}}(t) - \sigma_1\int_{t-\sigma_1}^{t} \dot{\bar{\beta}}^*(s)S_5\dot{\bar{\beta}}(s)ds - \sigma\int_{t-\sigma}^{t} \dot{\bar{\beta}}^*(s)S_6\dot{\bar{\beta}}(s)ds$$
$$+ \dot{\bar{\gamma}}^*(t)(\tau_1^2 S_7 + \tau^2 S_8)\dot{\bar{\gamma}}(t) - \tau_1\int_{t-\tau_1}^{t} \dot{\bar{\gamma}}^*(s)S_7\dot{\bar{\gamma}}(s)ds - \tau\int_{t-\tau}^{t} \dot{\bar{\gamma}}^*(s)S_8\dot{\bar{\gamma}}(s)ds. \quad (28)$$

$$\dot{V}_5(t) = \delta^2 f^*(\bar{\beta}(t))S_9 f(\bar{\beta}(t)) + h^2 g^*(\bar{\gamma}(t))S_{10}g(\bar{\gamma}(t))$$
$$- \delta(t)\int_{t-\delta(t)}^{t} f^*(\bar{\beta}(s))S_9 f(\bar{\beta}(s))ds - h(t)\int_{t-h(t)}^{t} g^*(\bar{\gamma}(s))S_{10}g(\bar{\gamma}(s))ds.$$

Using lemma 1, we obtain that

$$\dot{V}_5(t) \leq \delta^2 f^*(\bar{\beta}(t))S_9 f(\bar{\beta}(t)) + h^2 g^*(\bar{\gamma}(t))S_{10}g(\bar{\gamma}(t))$$
$$-\left(\int_{t-\delta(t)}^{t} f\left(\bar{\beta}(s)\right)ds\right)^* S_9\left(\int_{t-\delta(t)}^{t} f\left(\bar{\beta}(s)\right)ds\right)$$
$$-\left(\int_{t-h(t)}^{t} g(\bar{\gamma}(t))ds\right)^* S_{10}\left(\int_{t-h(t)}^{t} g(\bar{\gamma}(t))ds\right). \quad (29)$$

$$\dot{V}_6(t) = \delta_1^2\dot{\bar{\beta}}^*(t)S_{11}\dot{\bar{\beta}}(t) - \delta_1\int_{t-\delta_1}^{t} \dot{\bar{\beta}}^*(s)S_{11}\dot{\bar{\beta}}(s)ds + \delta^2\dot{\bar{\beta}}^*(t)S_{12}\dot{\bar{\beta}}(t)$$
$$- \delta\int_{t-\delta}^{t} \dot{\bar{\beta}}^*(s)S_{12}\dot{\bar{\beta}}(s)ds + h_1^2\dot{\bar{\gamma}}^*(t)S_{13}\dot{\bar{\gamma}}(t) - h_1\int_{t-h_1}^{t} \dot{\bar{\gamma}}^*(s)S_{13}\dot{\bar{\gamma}}(s)ds$$
$$+ h^2\dot{\bar{\gamma}}^*(t)S_{14}\dot{\bar{\gamma}}(t) - h\int_{t-h}^{t} \dot{\bar{\gamma}}^*(s)S_{14}\dot{\bar{\gamma}}(s)ds. \quad (30)$$

By using lemma 1 and lemma 2 that

$$-\sigma_1\int_{t-\sigma_1}^{t}\dot{\bar{\beta}}^*(s)S_5\dot{\bar{\beta}}(s)ds = -\sigma_1\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{\bar{\beta}}^*(s)S_5\dot{\bar{\beta}}(s)ds - \sigma_1\int_{t-\sigma_1(t)}^{t}\dot{\bar{\beta}}^*(s)S_5\dot{\bar{\beta}}(s)ds$$

$$\leq -\frac{\sigma_1}{\sigma_1-\sigma_1(t)}\left(\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{\bar{\beta}}(s)ds\right)^*S_5\left(\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{\bar{\beta}}(s)ds\right)$$

$$-\frac{\sigma_1}{\sigma_1(t)}\left(\int_{t-\sigma_1(t)}^{t}\dot{\bar{\beta}}(s)ds\right)^*S_5\left(\int_{t-\sigma_1(t)}^{t}\dot{\bar{\beta}}(s)ds\right)$$

$$= -\frac{\sigma_1}{\sigma_1-\sigma_1(t)}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}^*\begin{bmatrix}0\\ -I\\ I\end{bmatrix}S_5\begin{bmatrix}0\\ -I\\ I\end{bmatrix}^*\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}$$

$$-\frac{\sigma_1}{\sigma_1(t)}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}^*\begin{bmatrix}I\\ 0\\ -I\end{bmatrix}S_5\begin{bmatrix}I\\ 0\\ -I\end{bmatrix}^*\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}$$

$$\leq -\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}^*\begin{bmatrix}0 & I\\ -I & 0\\ I & -I\end{bmatrix}\begin{bmatrix}S_5 & U_1\\ U_1^* & S_5\end{bmatrix}\begin{bmatrix}0 & I\\ -I & 0\\ I & -I\end{bmatrix}^*\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}$$

$$= -\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix}^*\begin{bmatrix}S_5 & -U_1^* & -S_5+U_1^*\\ -U_1 & S_5 & -S_5+U_1\\ -S_5+U_1 & -S_5+U_1^* & 2S_5-U_1-U_1^*\end{bmatrix}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma_1)\\ \bar{\beta}(t-\sigma_1(t))\end{bmatrix} \quad (31)$$

Similarly,

$$-\sigma\int_{t-\sigma}^{t}\dot{\bar{\beta}}^*(s)S_6\dot{\bar{\beta}}(s)ds \leq$$

$$-\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma)\\ \bar{\beta}(t-\sigma(t))\end{bmatrix}^*\begin{bmatrix}S_6 & -U_2^* & -S_6+U_2^*\\ -U_2 & S_6 & -S_6+U_2\\ -S_6+U_2 & -S_6+U_2^* & 2S_6-U_2-U_2^*\end{bmatrix}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\sigma)\\ \bar{\beta}(t-\sigma(t))\end{bmatrix} \quad (32)$$

$$-\tau_1\int_{t-\tau_1}^{t}\dot{\bar{\gamma}}^*(s)S_7\dot{\bar{\gamma}}(s)ds \leq$$

$$-\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-\tau_1)\\ \bar{\gamma}(t-\tau_1(t))\end{bmatrix}^*\begin{bmatrix}S_7 & -U_3^* & -S_7+U_3^*\\ -U_3 & S_7 & -S_7+U_3\\ -S_7+U_3 & -S_7+U_3^* & 2S_7-U_3-U_3^*\end{bmatrix}\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-\tau_1)\\ \bar{\gamma}(t-\tau_1(t))\end{bmatrix} \quad (33)$$

$$-\tau\int_{t-\tau}^{t}\dot{\bar{\gamma}}^*(s)S_8\dot{\bar{\gamma}}(s)ds \leq$$

$$-\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-\tau)\\ \bar{\gamma}(t-\tau(t))\end{bmatrix}^{*}\begin{bmatrix} S_8 & -U_4^{*} & -S_8+U_4^{*}\\ -U_4 & S_8 & -S_8+U_4\\ -S_8+U_4 & -S_8+U_4^{*} & 2S_8-U_4-U_4^{*}\end{bmatrix}\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-\tau)\\ \bar{\gamma}(t-\tau(t))\end{bmatrix} \qquad (34)$$

$-\delta_1\int_{t-\delta_1}^{t}\dot{\bar{\beta}}^{*}(s)S_{11}\dot{\bar{\beta}}(s)ds \le$

$$-\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\delta_1)\\ \bar{\beta}(t-\delta_1(t))\end{bmatrix}^{*}\begin{bmatrix} S_{11} & -U_5^{*} & -S_{11}+U_5^{*}\\ -U_5 & S_{11} & -S_{11}+U_5\\ -S_{11}+U_5 & -S_{11}+U_5^{*} & 2S_{11}-U_5-U_5^{*}\end{bmatrix}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\delta_1)\\ \bar{\beta}(t-\delta_1(t))\end{bmatrix} \qquad (35)$$

$-\delta\int_{t-\delta}^{t}\dot{\bar{\beta}}^{*}(s)S_{12}\dot{\bar{\beta}}(s)ds \le$

$$-\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\delta)\\ \bar{\beta}(t-\delta(t))\end{bmatrix}^{*}\begin{bmatrix} S_{12} & -U_6^{*} & -S_{12}+U_6^{*}\\ -U_6 & S_{12} & -S_{11}+U_6\\ -S_{12}+U_6 & -S_{12}+U_6^{*} & 2S_{12}-U_6-U_6^{*}\end{bmatrix}\begin{bmatrix}\bar{\beta}(t)\\ \bar{\beta}(t-\delta)\\ \bar{\beta}(t-\delta(t))\end{bmatrix} \qquad (36)$$

$-h_1\int_{t-h_1}^{t}\dot{\bar{\gamma}}^{*}(s)S_{13}\dot{\bar{\gamma}}(s)ds \le$

$$-\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-h_1)\\ \bar{\gamma}(t-h_1(t))\end{bmatrix}^{*}\begin{bmatrix} S_{13} & -U_7^{*} & -S_{13}+U_7^{*}\\ -U_7 & S_{13} & -S_{13}+U_7\\ -S_{13}+U_7 & -S_{13}+U_7^{*} & 2S_{13}-U_7-U_7^{*}\end{bmatrix}\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-h_1)\\ \bar{\gamma}(t-h_1(t))\end{bmatrix} \qquad (37)$$

$-h\int_{t-h}^{t}\dot{\bar{\gamma}}^{*}(s)S_{1}\dot{\bar{\gamma}}(s)ds \le$

$$-\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-h)\\ \bar{\gamma}(t-h(t))\end{bmatrix}^{*}\begin{bmatrix} S_{14} & -U_8^{*} & -S_{14}+U_8^{*}\\ -U_8 & S_{1} & -S_{14}+U_8\\ -S_{14}+U_8 & -S_{14}+U_8^{*} & 2S_{14}-U_8-U_8^{*}\end{bmatrix}\begin{bmatrix}\bar{\gamma}(t)\\ \bar{\gamma}(t-h)\\ \bar{\gamma}(t-h(t))\end{bmatrix} \qquad (38)$$

The assumption (A) indicates that

$$0 \le \bar{\beta}^{*}(t)MR_1M\bar{\beta}(t) - f^{*}(\bar{\beta}(t))R_1 f(\bar{\beta}(t)), \qquad (39)$$

$$0 \leq \bar{\gamma}^*(t)NT_1N\bar{\gamma}(t) - g^*(\bar{\gamma}(t))T_1g(\bar{\gamma}(t)), \quad (40)$$

$$0 \leq \bar{\beta}^*(t - \sigma_1)MR_2M\bar{\beta}(t - \sigma_1) - f^*(\bar{\beta}(t - \sigma_1))R_2f(\bar{\beta}(t - \sigma_1)), \quad (41)$$

$$0 \leq \bar{\gamma}^*(t - \tau_1)NT_2N\bar{\gamma}(t - \tau_1) - g^*(\bar{\gamma}(t - \tau_1))T_2g(\bar{\gamma}(t - \tau_1)), \quad (42)$$

$$0 \leq \bar{\beta}^*(t - \sigma)MR_3M\bar{\beta}(t - \sigma) - f^*(\bar{\beta}(t - \sigma))R_3f(\bar{\beta}(t - \sigma)), \quad (43)$$

$$0 \leq \bar{\gamma}^*(t - \tau)NT_3N\bar{\gamma}(t - \tau) - g^*(\bar{\gamma}(t - \tau))T_3g(\bar{\gamma}(t - \tau)), \quad (44)$$

$$0 \leq \bar{\beta}^*(t - \sigma(t))MR_4M\bar{\beta}(t - \sigma(t)) - f^*(\bar{\beta}(t - \sigma(t)))R_4f(\bar{\beta}(t - \sigma(t))), \quad (45)$$

$$0 \leq \bar{\gamma}^*(t - \tau(t))NT_4N\bar{\gamma}(t - \tau(t)) - g^*(\bar{\gamma}(t - \tau(t)))T_4g(\bar{\gamma}(t - \tau(t))), \quad (46)$$

By following from (6), we get

$$0 = \left[\bar{\beta}(t) + \dot{\bar{\beta}}(t)\right]^* 2A[-\dot{\bar{\beta}}(t) + (-C + K_1)\bar{\beta}(t) + K_2\bar{\beta}(t - \sigma(t)) + K_3\bar{\beta}(t - \delta(t)) + W_1f(\bar{\beta}(t) + W_2f(\bar{\beta}(t - \sigma(t)) + W_3\int_{t-\delta(t)}^{t} f(\bar{\beta}(s))ds] \quad (47)$$

$$0 = [\bar{\gamma}(t) + \dot{\bar{\gamma}}(t)]^* 2B[-\dot{\bar{\gamma}}(t) + (-D + L_1)\bar{\gamma}(t) + L_2\bar{\gamma}(t - \tau(t)) + L_3\bar{\gamma}(t - h(t)) + Z_1g(\bar{\gamma}(t)) + Z_2g(\bar{\gamma}(t - \tau(t))) + Z_3\int_{t-h(t)}^{t} g(\bar{\gamma}(s))ds] \quad (48)$$

Formula (22) and (23) implies that $K_i = A^{-1}E_i$ and $L_i = B^{-1}F_i$, which together with inequalities (25) – (48) leads to

$$\dot{V}(t) \leq \xi^*(t) \ \theta \ \xi(t). \quad (49)$$

where

$\xi(t) = (\dot{\bar{\beta}}^*(t), \bar{\beta}^*(t), \bar{\beta}^*(t - \sigma_1), \bar{\beta}^*(t - \sigma), \bar{\beta}^*(t - \sigma_1(t)), \bar{\beta}^*(t - \sigma(t)), \int_{t-\sigma_1}^{t} \bar{\beta}^*(s)ds,$

$\int_{t-\sigma}^{t} \bar{\beta}^*(s)ds$, $f^*(\bar{\beta}(t))$, $f^*(\bar{\beta}(t - \sigma_1))$, $f^*(\bar{\beta}(t - \sigma))$, $\bar{\beta}(t - \delta_1)$, $\bar{\beta}(t - \delta_1(t))$,
$\bar{\beta}(t - \delta)$, $\bar{\beta}(t - \delta(t))$, $f^*(\bar{\beta}(t - \sigma(t)))$, $\int_{t-\delta(t)}^{t} f(\bar{\beta}(s))ds$, $\dot{\bar{\gamma}}^*(t)$, $\bar{\gamma}^*(t)$,
$\bar{\gamma}^*(t - \tau_1)$, $\bar{\gamma}^*(t - \tau)$, $\bar{\gamma}^*(t - \tau_1(t))$, $\bar{\gamma}^*(t - \tau(t))$, $\int_{t-\tau_1}^{t} \bar{\gamma}^*(s)ds$, $\int_{t-\tau}^{t} \bar{\gamma}^*(s)ds$,
$g^*(\bar{\gamma}(t))$, $g^*(\bar{\gamma}(t - \tau_1))$, $g^*(\bar{\gamma}(t - \tau))$, $\bar{\gamma}(t - h_1)$, $\bar{\gamma}(t - h_1(t))$, $\bar{\gamma}(t - h)$,
$\bar{\gamma}(t - h(t))$, $g^*(\bar{\gamma}(t - \tau(t))$, $\int_{t-h(t)}^{t} g^*(\bar{\gamma}(s))ds)^*$.

Hence, it can be derived from (21) and (49) that

$$\dot{V}(t) \leq 0.$$

which shows that the error dynamical system (6) is globally asymptotically stable. The proof is completed.

IV. NUMERICAL EXAMPLE

In this section, one numerical instance is supplied at the side of simulation results to demonstrate the ability advantages and effectiveness of the evolved technique for BAMCVNN with successive time-varying delays.

Example 1 Consider the BAM Complex-Valued Neural Networks (1) with the following parameters

$$C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix},\ D = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix},\ W_1 = \begin{bmatrix} 1.5 - 1.2i & 1.3 + 0.2i \\ 1.8 + 0.4i & 1.5 - 0.9i \end{bmatrix},$$
$$Z_1 = \begin{bmatrix} 1.7 + 0.2i & 1.2 - 0.9i \\ 1.5 - 0.6i & 0.5 + 1.1i \end{bmatrix}, W_2 = \begin{bmatrix} 1.9 - 0.4i & -0.3 - 0.2i \\ 0.8 + 0.9i & 1.5 + 0.5i \end{bmatrix},$$
$$Z_2 = \begin{bmatrix} 1 + 0.2i & -0.9 - 0.1i \\ 1.5 - 0.8i & 1.2 + 0.5i \end{bmatrix}, W_3 = \begin{bmatrix} 1.6 + 0.9i & 0.7 + 0.2i \\ 1.2 - 1.5i & 0.5 - 0.5i \end{bmatrix},$$
$$Z_3 = \begin{bmatrix} 1.7 - 0.3i & -0.5 - 1.7i \\ 1.2 + 0.8i & 1.5 + 0.4i \end{bmatrix}, I = \begin{bmatrix} \cos(0.3t) & (2\sin(t) - 1)i \\ sint(0.1t) & -(3\cos(0.5t) + 1)i \end{bmatrix},$$
$$J = \begin{bmatrix} 3\cos(0.1t) & -(\sin(2t) + 2)i \\ 2sint(t) & (3\cos(t) + 1)i \end{bmatrix}.$$

It is easy to check that the activation functions (A) are satisfied with

$$M = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

We assume that $\sigma_1 = \sigma_2 = 1.2$, $\delta_1 = \delta_2 = 0.4, \tau_1 = \tau_2 = 1$ and $h_1 = h_2 = 0.7$ by using MATLAB LMI control toolbox and by solving the LMIs in Theorem 3.1 in our paper. We conclude that the global synchronization of slave model (4) and the master model (1) is realized with the gain matrices

$$K_1 = A^{-1}E_1 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix},$$
$$K_2 = A^{-1}E_2 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix},$$
$$K_3 = A^{-1}E_3 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix},$$
$$L_1 = B^{-1}F_1 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix},$$
$$L_2 = B^{-1}F_2 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix},$$

$$L_3 = B^{-1}F_3 = \begin{bmatrix} 5.3589 + 0.0000i & -0.0033 - 1.1203i \\ -0.0033 - 1.1203i & 3.9964 + 0.0000i \end{bmatrix}.$$

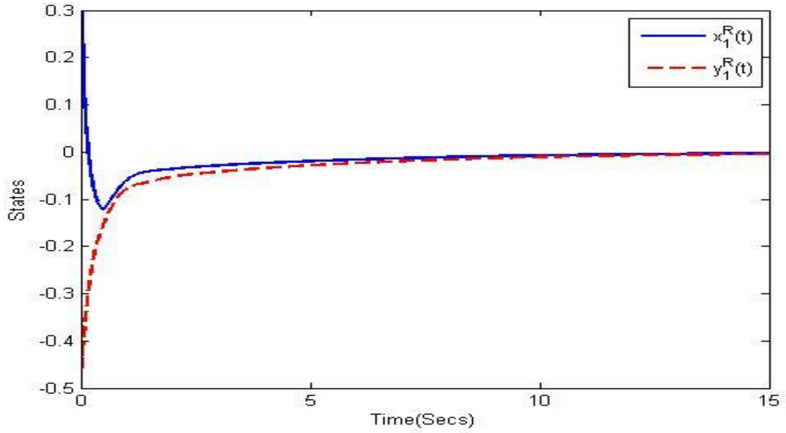


FIGURE-1

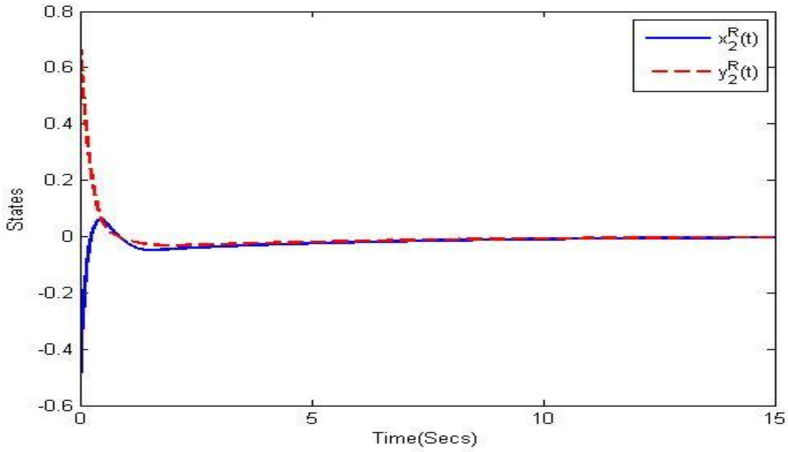
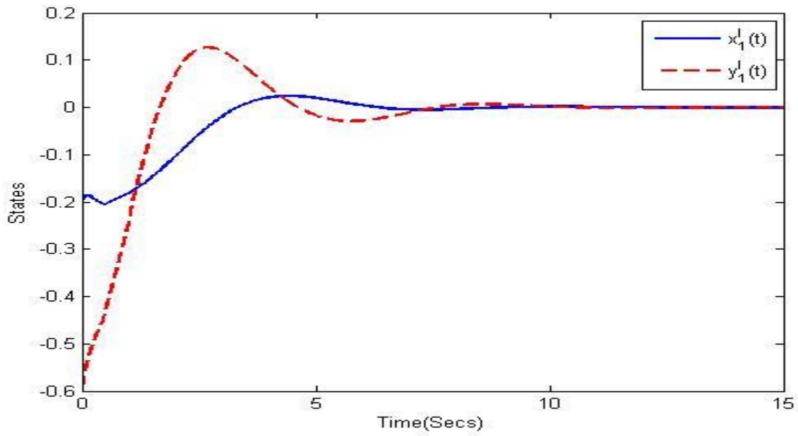


FIGURE-2

FIGURE-3



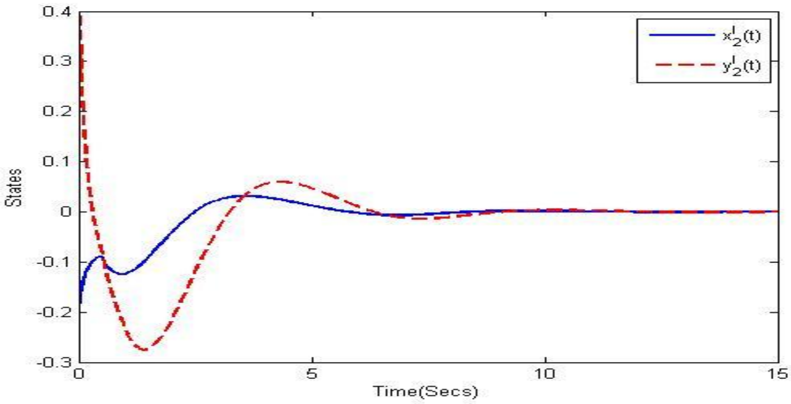


FIGURE-4

Figure 1 to **Figure 4** the real and imaginary parts of states for model (6) respectively.

IV.CONCLUSION

In this paper, the synchronization of the BAMCVNNs with both discrete and distributed successive time-varying delays is presented. A suitable augmented Lyapunov-Krasovskii functional (LKF) were constructed with more information about the activation functions and the upper bounds of the successive time-varying delays. Different from those traditional models, the considered BAMCVNNs are not divided into two real-valued NNs. Moreover, it's far really worth noting that the differentiability of the discrete and distributed successive time-varying delays isn't always required, that's extra popular than those in previous literature.

Novel delay-dependent the global synchronization for the master-slave NNs was derived in terms of LMIs with the aid of introducing a few zero fundamental inequalities and utilizing reciprocal convex combination technique. Finally, a numerical example is furnished to demonstrate and effectiveness of the proposed results.

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