Threshold Concepts in Undergraduate Mathematics Students' Problem-Solving Skills Development

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Abstract

This study aims to investigate some threshold concepts related to problem solving ability in mathematics of undergraduate mathematics students. Forty students were randomly selected through proportionate stratified sampling among the students majoring in mathin the upper three (3) curricular years. This investigation is a survey methodology using a correlational approach. Analysis of the data gathered yielded the following results: students' ability to solve verbal mathematics problems is dependent on his/her ability to translate verbal mathematics statements to symbols; reading comprehension is significantly related to the ability and skill in translating verbal mathematics statements to symbolic form and solving mathematics verbal problem; and students who have positive attitude toward verbal mathematics problem perform better than those with negative attitude. Thus, the basic skills on comprehension, translation of verbal statement to symbolic for and attitude of students are the threshold concepts needed toward building a strong foundation for successful problem-solving endeavor. Thus, teachers should be instrumental in the upliftment of the basic goal of education and should impart appropriate learning experiences in the teaching of problem solving.

Keywords: mathematics, verbal problem solving, comprehension, translation of verbal statement to symbol and attitude

Mathematicsperformance of students show that a good number get consistently high in numerical ability but notquite well with problem solving tests, particularly in verbal statement.Also, there are students who are performing well in other kinds of mathematics examination but not in problem solving test. When students are asked why they get low marks in verbal problem solving, their answer is: "I do not like verbal problem solving." If there is anything students would rather not to do in Math, it is problem solving (Ibe, 1988). What could be the reason for these situations.

Buchanan (1987) found that, attitude, motivation and belief systems are significant to performance. Also, Pimta, et. al., (2009) disclosed that some direct factors influencing mathematics problem-solving ability were attitude toward mathematics, self-esteem and teacher's teaching behavior while the indirect factors were motivation and self-efficacy.

Teachers of mathematics agree that games, contests, exercises, puzzles, enrichment activities, projects, videos, the use of modern tools, such as, computers, etc. are effective motivation for learning. But perhaps there is something else which can be related to the students' primary interest, such as reading comprehension. Concepcion (1971) concluded that

the general nature of the problem-solving difficulties of the students lies in their failure comprehend the problems in Mathematics. Furthermore, headds that their difficulty is compounded by language problems such as poor reading skills and inadequate vocabulary. The top-ranking cause of their errors in problem solving due to comprehension. Many schools have students who are creative particularly when it comes to their use of words.They write beautiful essays, short stories, poems and other forms of literary compositions. It is possible that learning how to solve problems in Mathematics can be as exciting as writing literary compositions when students can understand verbal problems. Thus, solving verbal mathematics problem and applying mathematics concepts by doing mathematical exercises would be as meaningful as an essay or short story. Then teachers will not hear students say, " I can solve a problem easily when written numerically or in the form of an equation, but when they are in plain English language they mean nothing to but me because I do not know how to start."
Ability to solve verbal problem is one of the most important objectives in the study of Mathematics. In fact, most teachers and mathematicians agree that to do Mathematics is to do problem solving (Belango&Carag, 2020). Perhaps seeing a mathematics verbal problem in the form of a paragraph with sentences, phrases, terms, numbers, symbols and other notations different from the terms used in other fields in difficult. Designating certain symbols, representing/translating some actual information about the problem is somewhat difficult and abstract. The presentation/translation of the verbal problem into symbols in the most important step in problem solving because it isa guide to the setting up of the correct equation. Meanwhile, the correctequation in the form of symbols connected with fundamental operations itself is the true representation of the entire verbal problem from which the solution of the problem can be generated by performing the indicated basic computations. Setting up the correct equation involves translating the verbal statement into algebraic language or symbolic notation in order to arrive at the solution of the problem. The skills in translating the verbal statement into algebraic language is determined by one's skill in reading, but many students are not very careful analytical readers. Furthermore, the difficulty encounteredby the students in interpreting/analyzing verbal statements of mathematical problems lies in the fact that these problems are not always stated explicitly but often implied and the students lack this skill to find out hidden are implications., Agustin (2021).

The above discussion is consistent with the second level of cognitive objective which is understanding or comprehension in the old version of the Bloom' s Taxonomy of Education Objectives. Some of the instructional objectives at this level are the ability to understand non-literal statements, skills in translating mathematical verbal material into symbolic statements and vice versa (Bloom et al.). Probably, one can translate the verbal problems into algebraic language or symbolic notation where he/she can write the correct equation, if he has the ability to understand the meaning of the problem in which include some hidden implications as the case may be. From the correct translation and the correct equation, one is already capable of producing the solution of the problem.

The levels of educational objectives require the individual to have knowledge which involves "the recall of methods and procedures; or the recall of pattern, structure, or setting." After this, the individual must have sufficient understanding/comprehension of the principles involve in the previously learned knowledge. Since education, is a continuous process, an individual should therefore undergo application of the theories, principles, methods, procedures and previously learned knowledge. All levels of educational objectives are mutually independent, that is, the occurrence of one level affects the occurrence of the other levels.

Therefore, more emphasis on some threshold concepts on the matters of comprehension, attitude toward mathematicsand ability to translate the verbal statement into symbols on the part of the students must be inculcated to prepare them to solve verbal problems. Although, there are many factors influencing mathematics problem solving ability but it would be more interesting if the initial concepts must be laid down first in order to establish a jumping rope toward building a strong foundation toward successful problem solving endeavor.

This study is intended to investigate some threshold concepts which are related tothe ability of the students to solve mathematics verbal problems. Specifically, it aims to determine whether

- ❖ student's ability to solve mathematics verbal problem dependent on his ability to translate verbal mathematical statements to symbol forms.
- ❖ The reading comprehension level is related to
  - o ability to translate verbal mathematical statement to symbolic form?
  - o ability to solve verbal mathematics problems?
- ❖ the ability of students who have positive attitude and negative attitude in mathematics verbal problem solving are different
- ❖ the grades of the students in Mathematics and English are related

Research Methodology

Research Design

Based on the nature of the hypothesis to be tested,this investigation is a correlational study using survey methodology. The subjects consisted of only one group wherein the conditions were taken as they were found in naturally occurring situations. The variables were mathematics verbal problem solving ability, reading comprehension level, ability to translate word expressions to symbolic forms and attitude of the students toward problem solving.

Sources error residing outside the subject were minimized by adopting a standard and reliable way of scoring the problem solving. This was done by letting three Mathematics teachers, the researcher included, ratecomplete solution of all problem together with the

presentation, identification of the given and the unknown, equation, computation, and the solution answer. After this process, the researcher considered the average number of points allocated by the three teachers in the different steps undertaken in problem solving.

For more revealing results all data gathering tools were administered four times. The total number of items were subdivided into four so that every subtest was administered separately with five days interval to insure reliability and validity of data gathered. Every subtest is one of the same nature and type but differ in some given conditions, unknown/s and other related information.

Description of the subjects

The subjects of the study consisted of a sample of 40 undergraduate students majoring inmathematics first year students of the College of Education of Isabela State University, Cabaganenrolled in school year, 2019-2020. Proportionate stratified random sampling was used on the subjects composing of a combination of boys and girls.

Collection of Data and Gathering Tools

Three kinds of test were administered to the subjects, namely: four sets of 10-item verbal mathematics problem solving tests, four sets of 10-item reading comprehension tests and four sets of 10-item identifying symbol translation tests. In addition to the 30-itemValidated Problem-solving Attitude Inventory was used in terms of content. Further, Cronbach's Alpha yieldeda reliability coefficient of 0.89 which is high. The results of the test were checked and scores were tabulated. Specifically, the attitude of the students were determined by reversing the order of the number code of the rate given by the students in the negative statements. After this process the researcher considered their total sum of the positive and negative statements as their attitude. As an incentive, the students were informed that the tests will have bearing on their Mathematics and English grades.

Before administering the mathematics verbal problem solving test and the identifying symbol translation test, the same were subjected for validation by competent Mathematics teachers of the Isabela State University, was Cabagan, likewise, the reading comprehension test validated by competent English teachers of the same school. The instruments were administered twice with one day interval to establish the reliability of the instruments.The following reliability coefficients were obtained of 0.87 (mathematics verbal problem solving test), 0.92 (identifying symbol translation test) and 0.90 (reading comprehension). Pretesting was conducted to those who were not involve in the study in the same school.

Statistical Tool

The Spearman Rank Correlation was used in computing the reliability of the instruments. The correlations were tested for significance using the t-test the significance of the coefficient correlation.

Pearson Product Moment Correlation used to determine whether the reading comprehension level and ability and skill in a) translating verbal mathematical statements to symbolic forms, and b) problem solving ability are positively correlated. Same was used to determine whether Mathematics and English grades are positively correlated. The t-test for the significance of the correlation coefficient was used.

The Chi-square test was also to determinewhether ability to solve verbal mathematics problems is dependent on the ability to translate verbal mathematics statements symbolic forms. The attitude of the students toward problem solving and their scores in problem solving were analyzed using the two-tailed t-test for independent means. The 0.05 level of significance was specified for all tests.

Discussion of Results and Findings

Table 2 strongly indicate that students who are more able to solve verbal mathematics problems have better facility in translating verbal mathematics statements to symbolic forms than are those less able to solve verbal problems. The proportion of the students achieving on a level of ability in problem solving differs from group to group, that is, the ability of the students to solve problems depends upon their abilities to translate verbal mathematics statements to symbolic forms. This is evident since the computed Chi-square value $x^2 = 28.91$ is greater than the tabular value $x^2 = 3.84$ at 5% level significance.Furthermore, it was found that it was significant at the .01 level. Hence, the ability of the students solve problems depends on their abilities to translate verbal mathematics statements to symbolic farm is highly probable. This finding supports Lualhati and Santiago (1982) that success in problem solving lies in translating word problem into its appropriate equation or mathematical symbol.

Table 2. Contingency Table Showing the Relationship betweenProblem Solving Ability and Mathematical Verbal Ability

Problem Solving Ability	Low	High	Total
Low	17 (8.55)	2 (10.45)	19
High	1 (9.45)	20 (11.55)	21
	18	22	40

Table 2, also proves that reading comprehension has something to do with ability to translate verbal mathematics statements to symbolic forms. This assumption is substantiated by the Pearson Product Moment Correlation Coefficient of 0.84, indicating a very high correlation between the two variables. This shows a strong positive correlation, that is, the as the scores of students in in reading comprehension increase or decrease, their scores ability in ability to translate verbal mathematics statements to t-test symbolic forms follow the same trend. Moreover, for the significance of the correlation coefficient has a computed value of $t = 11.42$ is greater than the tabular value $t = 1.960$. Thus, the correlation is significant at the

.05 level of significance. Further investigation indicates that it is highly significant even at the 0.01 level. Hence, the increase/decrease of the scores of the students in reading comprehension following the same trend in their scores on ability to translate verbal mathematics statements to symbolic forms is highly probable. For more revealing results, the coefficient of determination r^2= .7056 was determined, indicating that 70.56% the verbal mathematics statements to symbolic forms is accounted for by the scores in reading comprehension. This finding also supports (Bloom et al.) that one of the instructionalobjectives at the comprehension/understanding level is the skill in translating mathematical verbal material into symbolicstatements and vice versa.

Table 2. Correlation Value, t-value and Coefficient of Determination for Reading Comprehension and Ability to Translate Verbal Mathematics Statements

Variable	**Correlation Coefficient (r)**	**Description**	**t-Value**	**Coefficient of Determination**
Reading Comprehension VS Ability to Translate Verbal Mathematics Statements to Symbol	0.84	Very High Correlation	11.42*	70.56%

*** Significant at .01 level (two tailed test)**

Table 3 reveals that reading comprehension of a student and his/her ability to solve verbal mathematics problems are positively correlated. The Pearson Product Moment Correlation of 0.88 was obtained. In effect, as the scores of the students in reading comprehensionincrease/decrease, their scores in solving verbal mathematics problems follow the same trend. The t-test for testing the significance for correlation also supports this conclusion since the computed value: t = 11.97 is greater than the tabular value 1.960. Hence the correlation is highly significant event at the .01 level of significance. Evidently, the increase/decrease of the scores of the students in reading comprehension follow the same trend as their scores in problem solving. Further analysis using the coefficient of determination of r^2 = .7744 showed that 77.44% of ability in problem solving is accounted for by the scores in reading comprehension.

Table 3. Correlation Value, t-value and Coefficient of Determination for Reading Comprehension and Ability to Solve Verbal Mathematics Problem

Variable	**Correlation Coefficient (r)**	**Description**	**t-Value**	**Coefficient of Determination**
Reading Comprehension VS Ability to Solve Verbal Mathematics Problem	0.88	Very High Correlation	11.97*	77.44%

* **Significant at .01 level (two tailed test)**

This finding agrees with Concepcion (1971) who concluded that the general nature of the problem solving difficulties of the students lie in their failure to comprehend the problem mathematics.

The analysis of the data in Table 4 shows that students with positive attitudes (x_1= 76) and negative attitudes (x_2 = 56) differ in problem solving ability as shown by the t-test for independent groups. This is evident since the computed value, t = 4.37 falls in the critical region. The computed value is greater than the tabular value t = 1.960 which is significant at the level 0.01. The l researcher concludes that the students who have positive attitudes perform better in problem solving than those with negative attitude. Thus, the relationship between attitude and verbal problem solving ability seems warranted.

Table 4. Mean and t-Values of Positive and Negative Attitude Towards Mathematics

Variable	Mean	Computed t-Value
Positive Attitude	76	4.37*
Negative Attitude	56	

*** Significant at .01 level (two tailed)**

Using the data in Table 5, the Pearson Product Moment Correlation Coefficient reveals that the grades in Mathematics and English are not correlated as measured by the correlation coefficient of -0.07. This implies that Mathematics grade do not follow the same trend as the English grades. The absolute value of the correlation coefficient approaches zero, which means that the correlation is between two variables is negligible. This is confirmed the by t-test for the significance of the correlation. The computed t value: t =0.95 is less than the tabular value t= 1.96. Thus, the correlation is not significant.

Comment [H1]:

Table 5. Correlation Value, t-value and Coefficient of Determination for Reading Comprehension and Ability to Solve Verbal Mathematics Problem

Variable	**Correlation Coefficient (r)**	**Description**	**t-Value**
Reading Comprehension VS Ability to Solve Verbal Mathematics Problem	-0.07	Negligible	0.95*

Not significant at .05 level (two tailed test)

Conclusions

As a consequence, comprehension level can be associated with one's ability to solve problems, that is, a student who high comprehension also has better facility in solving mathematics problems. Since problem solving is complex process, getting the right solution is highly desirable and is should always result from a careful analysis of the problem. Some studies revealed that many students are not very careful analytical readers and are poor in interpreting/analyzing verbal problem mathematics. This lies in the fact that problems always stated explicitly but are often implied and students lack the skills to find out hidden implications. Tracing process, it suffices to note that very much related problem solving is one's ability to understand or comprehend the mathematics concepts and terms. Along this line, a student can conceptualize a problem better if his comprehension is sufficient. Moreover, coupled with this is the proper attitude of students toward problem solving.

There is sufficient evidence in this study, therefore to support the hypothesis that reading plays a key role in problem solving. The conclusions were drawn adequately supported since the reading comprehension and the ability to translate verbal mathematics statements to symbolic forms are positively correlated and the ability to solve verbal mathematics problem is dependent on the ability to translate verbal mathematics statements to symbolic forms. Also, reading comprehension is a requisite to mathematical verbal ability. At the same time, reading comprehension is requisite to problem solving and, likewise, to mathematical verbal ability. As a consequence, comprehension is of paramount importance to problem solving.

Thus, comprehension, ability to translate verbal mathematics statements to symbolic forms and attitude are threshold concepts or the beginning foundations developing success for problem solving.

Recommendations

Considering the importance of language in the understanding of verbal mathematics problems, the schools are expected to produce students who can communicate well in both Filipino and English Languages. This requires faculty upgrading particularly in the language and communication areas. In as much as English is the medium of instruction for mathematics, the students should acquire the necessary skills in the English language.

Since problem solving is the "heart" of mathematics, there may be a need to review the content of mathematics curriculum. The content curriculum is still relevant, additional exercises and topics should be made available for enrichment purposes to challenge the students in problem solving. Moreover, there may be a need to reorient teachers in the teaching of problem solving. Teachers/Instructors of mathematics should ne provided with the necessary training and professional development to allow them to cope with the demands of teaching problem solving.

The ability to generalize comes with the ability to analyze data. Translating data or information to equation requires mathematical orientation. There are skills which can be developed even at the elementary level. The skills of forming equation and translating mathematics verbal statement to symbolic form make solving mathematics problem easy. Teachers must therefore develop these skills in their students. This would require the training of teachers in their specific skills.

Considering that other fields like physics and chemistry rely heavily on mathematical concepts and principles. It is therefore essential that mathematics as a subject be taught properly (especially the topic problem solving) by teachers and be appreciated by the students, especially at the school level, (Malana,2020). Too often, as propounded by Malana (2020) many college science faculty complain of poor student performance in physics and chemistry which they attributed to poor student background in mathematics. Hence, high school subjects in mathematics, including foundation courses at the tertiary level, should be strengthened if we expect more students to elect science careers.

Teachers should work hand in hand to find out the factors associated with the negative attitude of students toward problem solving.

Considering the limitations of this study, a similar investigation should be made in order to ascertain the validity of the findings. If possible, a bigger population or a regional/ national study with wide scope should be used.

Implications

Many authors agree that a basic purpose in teaching mathematicsis to develop problem-solving behavior which is to prepare the individuals for numerous problems which arise everyday, especially in the complex society we live in. Since teachers have a direct contact with student, they must be instrumental in the upliftment of the basic goal education

and that they should impart appropriate learning experiences in the teaching of problem solving.

In order to achieve this, threshold concepts as door steps toward mathematics verbal problem solving should be strengthened. In view of the findings of this study, teachers and learners ought to meet the need for better teaching and learning problem solving, that is the identified threshold concepts as requisites should be properly taught to the students so as to build a strong foundation with the end and view to raise students' achievement in mathematics. Review of the content of mathematics curriculum and faculty upgrading in necessary inorder to achieve quality education.

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